LPP 6: Framing

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A Company manufactures two types of articles A and B. The contribution for each article as calculated by the accounting department is Rs. 50 per A and Rs. 70 per B. Both products are processed on three machines M1 M2 and M3. The time required (in hours) by each article and the total time available per week on each machine are as follows. How should the manufacturer schedule his production in order to maximize contribution? Formulate the problem as a LPP.

Machine	A	B	Available hours per Week
M1	3	2	46
M2	5	2	60
M3	2	6	70

Let x and y be the no. of units of articles A and B produced respectively.

1 unit of article A gives Rs 50 as the contribution.

$\therefore$ x units of article A will give Rs $50x$ as contribution.

Similarly,

y units of article B will gives Rs $70y$ as the contribution.

Total contribution = Rs $(50x + 70y)$

Let $Z = 50x + 70y$

$\because$ We are required to maximize the contribution,

Objective function can be given as,

Max $Z = 50x + 70y$.

To produce 1 unit of article A, it requires 3hrs on M_1

so to produce x units, " A, it will require $3x$ hrs on M_1

To produce 1 unit of article B, it requires 2hrs on M_1,

so to produce y units, " ", it will require $2y$ hrs on M_1

Total time required on machine M_1 is

$(3x + 2y)$ hrs.

But max. available time to operate M_1 per week is 46hrs

so

$$3x + 2y \not> 46$$

$$\Rightarrow 3x + 2y \leq 46 \rightarrow M_1$$

Constraints with regard to machine M_2 and M_3 are

$$5x + 2y \leq 60 \rightarrow M_2$$

$$2x + 6y \leq 70 \rightarrow M_3$$

$\because$ The no. of units produced cannot is negative,

so, $x \nless 0$, $y \nless 0$

$\Rightarrow x \geq 0$, $y \geq 0$

Entire LPP can be given as follows:

Max $Z = 50x + 70y$

Subject to constraints: $3x + 2y \leq 46$, $5x + 2y \leq 60$, $2x + 6y \leq 70$

Non-negativity constraints: $x \geq 0$, $y \geq 0$.

A diet conscious housewife wishes to ensure certain minimum intake of vitamins A, B, C for the family. The minimum daily needs of the vitamins A, B, C for the family are respectively 30, 20 and 16 units. For the supply of these minimum vitamin requirements, the housewife relies on two fresh foods. F1 and F2 . The F1 provides 7,5,2 units of the three vitamins per gram respectively and the F2 provides 2,4, 8 units of the same three vitamins per gram of the foodstuff respectively. F1 costs Rs. 3 per gram and F2 costs Rs. 2 per gram. Formulate a LPP to minimize the daily cost of Food stuff F1 and F2.

Min Reqd.

$\geq$

Vitamins	F1	F2	Min. Requirement of Vitamins
A	7	2	30
B	5	4	20
C	2	8	16

Let x gms and y gms be the daily requirement of food F_1 and F_2 respectively.

1 gm of food F_1 costs Rs 3
So x gms of food F_1 will cost Rs $3x$.
1 gm of food F_2 costs Rs 2
$\therefore$ y gms of food F_2 will cost Rs $2y$

Thus the total cost on x gms of F_1 and y gms of F_2 is Rs $(3x+2y)$.
Let $Z = 3x+2y$.

∴ we are required to minimize the cost of food, objective function can be given as:

$$\text{Min } Z = 3x + 2y$$

Vitamin A

1 gm of food F_1 has 7 units of vitamin A
∴ x gms of food F_1 will have $7x$ units of vitamin A.
1 gm of food F_2 has 2 units of vitamin A
∴ y gms of food F_2 will have $2y$ units of vitamin A

So the total content of vitamin A from food F_1 and F_2 is $(7x+2y)$ units.
But the minimum requirement of vitamin A for the family is 30 units daily.
So $7x + 2y \nless 30$

$$\Rightarrow 7x + 2y \geq 30 \rightarrow A$$

similarly we get,
$$5x + 4y \geq 20 \rightarrow B$$
$$2x + 8y \geq 16 \rightarrow C$$

∵ The quantity of food stuff consumed cannot be negatively

ie $x \nless 0, \quad y \nless 0$
$\Rightarrow x \geq 0, \quad y \geq 0$

Thus the entire LPP can be given as follows:

objective function : $\text{Min } Z = 3x + 2y$

Subject to constraints : $7x + 2y \geq 30$
$5x + 4y \geq 20$
$2x + 8y \geq 16$.

Non-negatively constraints $x \geq 0, \; y \geq 0$

The MNS Company has been a producer of picture tubes for television sets and certain printed circuits for radios. The company has just expanded into full scale production and marketing of AM and AM-FM radios. It has built a new plant that can operate 48 hours per week. Production of an AM radio in the new plant will require 2 hours and production of an AM-FM radio will require 3 hours. Each AM radio will contribute Rs. 40 to profits while an AM-FM radio will contribute Rs. 80 to profits. The marketing department, after extensive research has determined that a maximum of 15 AM radios and 10 AM-FM radios can be sold each week. Formulate the optimum production mix of AM and AM-FM radios that will maximize profits.

Let x and y be the no. of units of AM and AM-FM radios respectively.

Profit on 1 unit of AM radio is Rs 40
so profit on x units of AM radios will be Rs $40x$.
Profit on 1 unit of AM-FM radio is Rs 80
so profit on y units of AM-FM radios will be Rs $80y$.
Total profit on x AM radios and y AM-FM radios will be

Rs $(40x + 80y)$

Let $Z = 40x + 80y$

Thus the objective function can be given by,

$\text{Max } Z = 40x + 80y$.

To produce 1 unit of AM-radio, time required is 2 hrs.
So to produce x units of " ", time required will be $2x$ hrs
To produce 1 unit of AM-FM radio, time required is 3 hrs
so to produce y units of AM-FM radios, time required will be $3y$ hrs

Total time required to produce x units of AM radios and y units of AM-FM radios, will be

$(2x + 3y)$ hrs

But the production unit can be operated for at most 48 hrs a week.

Thus, $2x + 3y \not> 48$

$\Rightarrow 2x + 3y \leq 48$. — (1)

Max of 15 units of AM radios can be sold per weeks

$x \not> 15$

$\Rightarrow \boxed{x \leq 15}$ — (2)

Similarly, $\boxed{y \leq 10}$ — (3)

$\because$ The no. of units produced cannot be negative,

ie $x \not< 0$, $y \not< 0$.

$\Rightarrow x \geq 0$, $y \geq 0$

Thus the entire LPP can be given as follows;

Objective function : $\text{Max } Z = 40x + 80y$

Subject to constraints: $2x + 3y \leq 48$, $x \leq 15$, $y \leq 10$, $x \geq 0$, $y \geq 0$.

The standard weight of a special purpose brick has to be at least 5kgs and has to contain two basic ingredients B1 and B2. B1 costs Rs. 5 per kg and B2 cost Rs. 8 per kg. Strength consideration dictate that the brick contains not more than 4kgs of B1 and a minimum of 2kgs of B2. Formulate a LPP to minimize the cost of brick satisfying the above conditions.

$\text{Min } Z = 5x + 8y$

Soln :-

B_1

$x \not> 4$

$\Rightarrow x \leq 4$ — (1)

B_2

$y \not< 2$

$\Rightarrow y \geq 2$ — (2)

$x + y \not< 5$

$\Rightarrow x + y \geq 5$ — (3)

$\because$ The wts. of ingredients B_1 and B_2 cannot be negative

ie, $x \not< 0$, $y \not< 0$

$\Rightarrow x \geq 0, \quad y \geq 0$

— x —

Let x Kgs and y Kgs be the quantity of ingredient B_1 and B_2 respectively.

Cost of 1 Kg of B_1 is Rs 5
so cost of x Kgs of B_1 will be Rs $5x$
Cost of 1 Kg of B_2 is Rs 8
so cost of y Kgs of B_2 will be Rs $8y$.
Thus the total will be Rs $(5x + 8y)$
Let $Z = 5x + 8y$.
$\because$ We are required to minimize the cost of brick, the objective function can be given as,
Min $Z = 5x + 8y$.
Total wt of the brick in terms of ingredients B_1 and B_2 is $(x + y)$ Kgs
But ~~&~~ weight of brick has to be at least 5 Kgs

$$x + y \not< 5$$

$$\Rightarrow \boxed{x + y \geq 5} \quad — (1)$$

wt. of B_1 is x Kgs, and it cannot be more than 4 Kgs.

$$x \not> 4$$

$$\Rightarrow \boxed{x \leq 4} \quad — (2)$$

wt of B_2 is y Kg, and it has to be (at least) minimum 2 Kgs

$$y \not< 2$$

$$\boxed{y \geq 2} \quad — (3)$$

As the quantity of ingredients B_1 and B_2 cannot negative.

ie, $x \not< 0, \quad y \not< 0$
$\Rightarrow x \geq 0, \quad y \geq 0$.

Thus the entire LPP can be given as follows:
Objective function : Min $Z = 5x + 8y$
Subject to constraints : $x + y \geq 5, \; x \leq 4, \; y \geq 2$

Non-negativity constraints: $x \geq 0, \; y \geq 0$

(7) soln:-

Resources	A	B	Max. Availability
Plywood	2	1	20
Sunmica	1	3	15

Profit on 1 unit of A is Rs 30

Profit on 1 unit of A is Rs 30
" " 1 unit of B is Rs 20

Let x and y be the no. of units of Product A and B respectively.
Profit on 1 unit of Product A is Rs 30.
So profit on x units of product A will Rs $30x$.
Profit on 1 unit of product B is Rs 20.
So profit on y units of " " will be Rs $20y$.
Thus the total profit will be Rs $(30x + 20y)$
Let $Z = 30x + 20y$.
∴ We are required to maximize the profit,
Objective function can be given as,

$$\text{Max } Z = 30x + 20y$$

1 unit of product A, requires 2 sq. metres of plywood.
So x units " " ", will require $2x$ sq. metres of plywood.
1 unit of product B, requires 1 sq. metre of plywood.
So y units of " ", will require y sq. metres of plywood

So total quantity of plywood required to produce x units of A and y units of B is

$$(2x + y) \text{ sq. metres}$$

But maximum availability of plywood is 20 sq. metres.

$$2x + y \ngtr 20$$

$$\Rightarrow \boxed{2x + y \leq 20} \quad — (1)$$

Similarly, we get

$$x + 3y \leq 15 \quad — (2)$$

∵ the no. of units produced cannot be negative;

$$\text{ie} \quad x \nless 0, \quad y \nless 0$$

$$\Rightarrow x \geq 0, \quad y \geq 0$$